



Sample Session A

Pac(k)-man



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How can I use the sample sessions?

- Our sample sessions are designed as a taster for Axiom Maths circles. Use them with mathematicians that are interested in participating but seeking more information about the programme before committing.
 - The content and structure is representative of Axiom Maths sessions.
 - The speech bubbles (hidden in presentation view) are included to point out common features of Axiom Maths sessions.
- Sample sessions are designed to be useable with larger groups or whole classes at the beginning of Year 7.
- Training materials that accompany the sample sessions can be found [here](#).

The ABC of mentoring



Advance plan

- Tackle the problems yourself and experience the productive struggle
- Consider what the mathematicians will do, say and ask
- Read through the mentor notes and watch the accompanying videos

What are the misconceptions?

What's the same,
what's different?

What do you
notice?

Bring out the maths from them

- Ask questions to extend their mathematical thinking
- Introduce representations to expose the mathematical structure
- ...but give them space to be adventurous. Pause, wait and watch what they do. Get everyone used to an unawkward silence.



Catalyse conversations

- Sometimes individually, sometimes collectively
- Model and ask for precise mathematical vocabulary
- Help them to help each other
- No child left behind on the mountainside

Are you sure?
Convince me.

Agree or
challenge?

What is the correct
mathematical term for that?



Modes of engagement

In your '**advance planning**', decide how your circle will engage with the problems so that you '**bring out the maths from them**' and '**catalyse conversations**'. You should consider many things, including the set up of your circle (e.g. online/in person). However, Axiom Maths recommends varying the modes of engagement so that the mathematicians learn to write, think and speak mathematics, both independently and collaboratively.

Mode	Description	Icon
Independent	The mathematicians grapple with the problems individually, recording their own efforts. The mentor engages with the mathematicians individually. The mathematicians compare methods with each other as they work.	
Collaborative	The mathematicians work together towards a solution, and create a shared record of their efforts.	
Discussion	The mentor catalyses a conversation between the mathematicians about work that they have done, or are about to do.	
Quick fire	The mathematicians are time limited in preparing a response to a problem, and show their responses to the mentor and circle simultaneously (e.g. on mini-whiteboards/ in an online 'chat blast')	



The intention of the session

- The problems in this session aim to bring out the distinction between “I can’t do it” and “No-one can do it, it’s impossible.” Ask questions such as, “How can you **convince me** that this won’t work whatever way you try to fit the parcels in?”
- The mathematicians are introduced to a new term, conditioning, which here describes dividing up the arrangements according to where a specific parcel could go. This method helps them to search systematically, rather than in an undirected way, for all the solutions to the problems.



Overview of the session

In this **sample** session, we suggest you may finish the first three 'core' problems in one hour. If you have more or less than this time, use the additional problem for 'rapid graspers' and the guidance below to organise and structure your taster session(s) as you see fit.

Problem	Guidance	You may need...
Pac(k)-man	Introduces the post office context with an impossible problem.	Worksheets or Polypad
Sort it out	Use <u>conditioning</u> to find and justify how many ways the parcels can be placed in the sacks.	
True or not true?	A shorter problem in which the mathematicians will decide if two claims are true.	
A puzzle for Pac(k)-man	Additional problem for those that progress quickly.	

For some problems we provide worksheets or online tools.



What is a maths circle?

A maths circle is a group of pupils who gather with a mentor to be mathematicians together!

In our **maths circle** we

- **grapple** with (but not always solve!) intriguing problems and ideas
- learn to think, speak and work like a **university mathematician**
- sometimes think on **our own**, sometimes think **in a team**
- enjoy getting **stuck** as well as getting **unstuck**
- have **fun** ourselves, and make it fun for everyone else too!



Reinforce that they will be working on maths problems *together*. In maths circles it's not about competing against each other!

Good mathematicians are...



Adventurous



Articulate



Accepting

In maths circles you'll learn to be **adventurous**, **articulate** and **accepting** mathematicians!

The first three sessions in Block 1 each focus on one of the three **AAA** behaviours of mathematicians.

Pac(k)-man

In a post office, parcels are being placed into sacks.
No sack is allowed to contain **more than 10 kg** of parcels.

A postal worker has eight parcels which weigh the following:



They only have **three** sacks left.
Can they fit them into the three sacks?

Contexts in Axiom Maths problems are simple and familiar. The context is chosen to make it easier for the mathematicians to remember what they are trying to achieve.

Pac(k)-man

Look and listen for...

- The mathematicians **articulating** their strategy to either:
 - Fit as many parcels into the sacks as possible.
 - “I started with three parcels in the first sack, the two 2 kgs and the 3 kg. Then I fitted two parcels in the second sack, the two 4 kgs or the 4 kg and the 5 kg. But 4 kg, 7 kg and either 4 kg or 5 kg parcels were left and in total they’re too heavy!”
 - Follow with “What’s the problem with this approach?” and listen for “I didn’t *fill* each sack completely. I could fit more weight in each sack, so this was not an efficient approach.”
 - Or fit as much weight into the sacks as possible.
 - “I started by finding some parcels which together weigh exactly 10 kg. I chose the 7 kg and the 3 kg. Then I chose 4 kg, 4 kg and 2 kg. But 5 kg, 4 kg, 2 kg were left and in total they’re too heavy!”
 - Follow with “Why was this an efficient approach?”
- “Hang on, wait... they cannot possibly fit!” Ask the mathematicians to **convince you** and listen for “The total weight is more than 30 kg!”.

The ‘Look and listen for...’ slides outline what you might expect to see or hear from your mathematicians.

Words shown in **purple bold** are the AAA behaviours and Axiom Maths’ key phrases such as “Convince me”.



Pac(k)-man

Bring out the maths from them

- **This problem is impossible. The purpose is for the mathematicians to *justify why the problem is impossible*.** Don't rush them to this conclusion. Give them time to be **adventurous** independently first. 
- After enough thinking time to understand the context, suggest the mathematicians work in pairs or threes. 
- Use shared mini-whiteboards or our interactive Polypad to encourage collaboration.
- **Bring out the distinction between “I can't do it” and “No-one can do it, it's impossible”.**
 - If the mathematicians say, “This is impossible” because they can't do it, ask “How can you be *sure* that no one else will be able to say how the worker can fit the parcels in the sacks?”
 - If they convince themselves that it isn't possible by one example e.g. “I tried it once and it didn't work, so it can't be done”, say “I think you should try another way.” Let them do so, then ask “Can you **convince me** that this won't work whatever way you try to fit the parcels in?”
- **Before moving on, come back together as a circle to reinforce why this is impossible: The total weight is more than 30 kg.**

These slides show the essential learning in **bold** and suggest ways to 'Bring out the maths' from the mathematicians. Further prompts are provided that you might find useful.



Sort it out

In a post office, parcels are being placed into sacks.
No sack is allowed to contain **more than 10 kg** of parcels.

a)

9 kg	7 kg	6 kg
4 kg	3 kg	1 kg

b)

9 kg	7 kg	5 kg
4 kg	2 kg	2 kg

Part c) appears on the next slide, so mentors can decide when it appears to their mathematicians. This might be straight away or after they've tried a) and b).

Shown above are the weights of the parcels that the postal worker needs to fit into three sacks on two different days.

In each of the cases say whether the postal worker

- can fit the parcels into only three sacks in **only one** way,
- can fit the parcels into only three sacks in **more than one** way,
- or **cannot fit** the parcels in the sacks at all.

Axiom Maths circles are carefully sequenced: this question will help prepare the mathematicians for a more formal treatment of combinatorics in later sessions.

Sort it out

Look and listen for...

- For a)
- “I can see how to use number bonds to 10.”
- “Because the total is 30 kg, there has to be 10 kg in each sack.”
- “There’s only one way to do it.
9 kg and 1 kg together,
7 kg and 3 kg together,
6 kg and 4 kg together.”

Asking for the mathematicians’ reasoning is what effective mentors do throughout each session.

Bring out the maths from them

- Decide how the mathematicians will communicate their justifications for each part: will they write down a sentence on paper, or in the online chat (if working online), or speak ‘round the table’ one at a time?
- For these problems, first ask them to work independently. Speak to them individually to ‘sense check’ their reasoning, then bring them together to compare and comment on each other’s work.
- **For a) bring out that there is exactly one way.**
 - Ask “Could any of the sacks contain less than 10 kg?”
 - “Is there another way of fitting the parcels in the sacks? How do you know?”



Look for these icons in the guidance for suggested ‘modes of engagement’.



Sort it out

Look and listen for...

For b)

- The mathematicians incorrectly assuming the inverse of the conclusion from a) holds, “The total weight is only 29 kg, so it *must* be possible.”
- “The total is 29 kg, so I don’t know straight away that it’s impossible, but after trying it out, I’m convinced that it must be.”
- The mathematicians explaining their reasoning, using words such as “must” or “cannot”. For example:
 - “9 kg must go on its own.”
 - “7 kg can only go with one of the 2 kg parcels.”
 - “Then the 5 kg can go with either the 4 kg or the remaining 2 kg.”
 - “But that leaves me with one parcel that cannot go in a sack.”

Bring out the maths from them

For b)

- **Bring out that although the total weight is 29 kg, you can’t assume it *must* be possible to fit the parcels into three sacks. Ask them why this is.**
- Like in the previous problem, ‘Pac(k)-man’ the distinction between “I can’t do it” and “No-one can do it, it’s impossible” is important to reinforce here. Bring this out by asking:
 - “What arrangements have you tried? How can you be *sure* that no other arrangement will work?”
 - “Are you convinced that it’s impossible? Why? How will you **convince me?**”



Sort it out

In a post office, parcels are being placed into sacks.
No sack is allowed to contain **more than 10 kg** of parcels.

a)

9 kg	7 kg	6 kg
4 kg	3 kg	1 kg

b)

9 kg	7 kg	5 kg
4 kg	2 kg	2 kg

c)

7 kg	6 kg	4 kg	4 kg
3 kg	2 kg	2 kg	1 kg

Shown above are the weights of the parcels that the postal worker needs to fit into three sacks on three different days.

In each of the cases say whether the postal worker

- can fit the parcels into only three sacks in **only one** way,
- can fit the parcels into only three sacks in **more than one** way,
- or **cannot fit** the parcels in the sacks at all.

Sort it out

Look and listen for...

For c)

- The mathematicians using a “swapping” strategy to show that there is more than one way.
 - For example: “I’ve found one way: $7 + \underline{3}$, $6 + 4$, $4 + 2 + \underline{2 + 1}$.”
 - “Because the 3 kg is in one sack and the 2 kg and 1 kg are in another sack together, I can swap them to find another way: $7 + \underline{2 + 1}$, $6 + 4$, $4 + 2 + \underline{3}$.”
- Or by using the total weight of 29 kg to reason that there can’t be a sack with less than 9 kg and then writing down all the possibilities of parcels that can go with 7 kg. Listen for
 - “There can’t be a sack with less than 9 kg so the 7 kg can go with the 2 kg ($7 + 2$) or with the 3 kg ($7 + 3$) or with the 1 kg and the 2 kg together ($7 + 1 + 2$).”
 - And then reasoning there is more than one way. For example,
“Let’s say 7 kg is with the 2 kg ($7 + 2$)
then the 6 kg can go with the 4 kg ($6 + 4$)
or the 6 kg can go with the 3 kg and 1 kg ($6 + 3 + 1$).”
- “... I’ve found all seven possibilities.” Look for

$7 + 2$,	$6 + 4$,	$4 + 3 + 2 + 1$
$7 + 2$,	$6 + 3 + 1$,	$4 + 4 + 2$
$7 + 1 + 2$,	$6 + 4$,	$4 + 3 + 2$
$7 + 1 + 2$,	$6 + 3$,	$4 + 4 + 2$

$7 + 3$,	$6 + 2 + 1$,	$4 + 4 + 2$
$7 + 3$,	$6 + 4$,	$4 + 2 + 2 + 1$
$7 + 3$,	$6 + 2 + 2$,	$4 + 4 + 1$



Sort it out

Bring out the maths from them

For c)

- After one arrangement has been found, ask “Is there another way of fitting the parcels in the sacks? And another? How many ways can you find?”
- **The intention of c) is to promote a systematic approach to finding more than one way. Support them with this and introduce the word conditioning. Note that this terminology is reintroduced in Block 1 of the programme.**
 - Ask “Where do you want to put the heaviest two parcels?” Listen for the mathematicians agreeing that the 7 kg and 6 kg must fit in separate sacks.
 - Follow with “Where can you put the 4 kg parcels?” Listen for the two possibilities and then show the next slide to support the next part of the argument.
 - Then ask “If you put them in here [choose one of the two sacks], where can the other parcels go?” and then “Now let’s consider choosing the other sack.” Say that this is called conditioning on where the 4 kg parcels go.
 - Then reinforce the language of conditioning and support the mathematicians to use it to explain their methods. Ask “What are you conditioning on now?” For example, listen for “I’ve placed the 4 kg, 6 kg and 7 kg parcels, and now I’m considering what happens when I place the 3 kg parcel in each of the different sacks that it can go. I am conditioning on where the 3 kg parcel goes.”

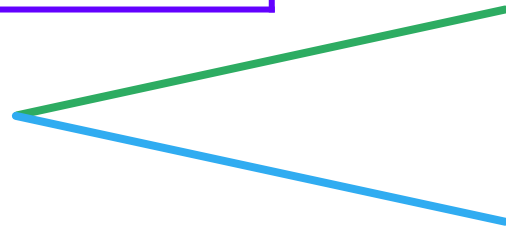


Sort it out

In a post office, parcels are being placed into sacks.
No sack is allowed to contain **more than 10 kg** of parcels.

c)

~~7 kg~~	~~6 kg~~	~~4 kg~~	~~4 kg~~
3 kg	2 kg	2 kg	1 kg



7 kg

6 kg
4 kg

4 kg

7 kg

6 kg

4 kg
4 kg

In each case, where can the other parcels go?
This is called **conditioning** on where the 4 kg parcels go.

True or not true?

In a post office, parcels are being placed into sacks.
No sack is allowed to contain **more than 10 kg** of parcels.

Is **either** or **both** of these claims true?

- a) **If** the parcels can be placed in three sacks
then their total weight is at most 30 kg.
- b) **If** the total weight of the parcels is at most 30 kg
then they can be placed in three sacks.

True or not true?

Look and listen for...

For a)

- “The first claim is always true. Each sack has to contain 10 kg or less. I can’t fit more than 30 kg of parcels into three sacks.”

For b)

- “But the second claim isn’t true. In b) the total weight is less than 30 kg, but the parcels don’t fit into three sacks. There are too many heavy parcels and not enough light ones.”

Bring out the maths from them

- Encourage the mathematicians to use the previous examples to help them to justify their reasoning to one another. If they come up with their own, celebrate their **adventurous** behaviour. 
- For a)
 - If they say the first claim is *always* true because “In a) and c), I can fit the parcels in three sacks and in both the total is 30 kg or less”, say that you agree but these are specific examples.
 - Ask “Can you **convince me** the first claim is *always* true?”
- For b)
 - “Which example shows that the second claim is not true?”
 - **Ask why they only need to use one example to be sure the second claim is not true, but they *cannot* use only one example to be sure the first claim is always true.**



Today, you have you been...



Adventurous

I kept trying to fit the parcels in the sacks even after my first idea didn't work.



Articulate

I convinced my partner why the parcels couldn't fit in the sack.



Accepting

I used my partner's solution to find another way to fit the parcels in the sack.

At the end of each Axiom Maths session (as well as throughout), the mathematicians reflect on how they have shown **AAA** behaviour!

Today, you have you been...

Bring out the maths from them

- **Allow 2 minutes at the end of the session to tell the mathematicians how they have demonstrated the three AAA behaviours of mathematicians today.**
- Ask them to reflect on what they enjoyed about this session, and how it might have been different from a normal maths lesson.
- “Is there somebody who you think has been especially **adventurous/articulate/accepting** today? What did they do?”

The first three sessions in Block 1 each focus on one of the three **AAA** behaviours of mathematicians.



Problems for rapid graspers

A puzzle for Pac(k)-man

Parcels are being placed into sacks in a post office.

Each of the parcels is a **whole number** of kilograms, and each parcel weighs **no more than 10 kg**.

Can you find a collection of

- a) **9 parcels** that weigh **exactly 28 kg** in total and don't fit in **three** sacks?
- b) or **10 parcels** that weigh **no more than 40 kg** in total and don't fit in **four** sacks?

The green-themed problems are for rapid graspers. These problems are not part of the core curriculum, and so future problems will not build on these concepts.

A puzzle for Pac(k)-man

Look and listen for...

For a)

- A correct and justified example, such as 6 kg, 6 kg, 6 kg, 5 kg, 1 kg, 1 kg, 1 kg, 1 kg, 1 kg with the reasoning that “Each of the 6 kg parcels needs to go in a separate sack, so each of the sacks will have a 6 kg parcel. Once these are placed, there is nowhere to place the 5 kg parcel because $6 + 5 = 11$.”

For b)

- A correct and justified example such as:
 - “Five 6 kg parcels and five 1 kg parcels. Each sack can have only one 6 kg parcel, but there are five 6 kg parcels and four sacks.”
 - Or “Four 2 kgs, two 3 kgs, a 5 kg, and three 7 kgs. The total weight is 40 kg, but I can only fit at most 29 kg into three sacks ($7 + 3$, $7 + 3$ and $7 + 2$) so I’m left with 11 kg to try to fit into the fourth sack. I can’t!”

Bring out the maths from them

- This is an extended problem where the mathematicians can be creative – something that happens frequently in Axiom Maths circles but might seem unusual to them! Here, they are asked to create their own examples to fit some given criteria.
- **Provide opportunities for the mathematicians to be accepting.** When one of your mathematicians has created an example for either of the two scenarios, ask them to share it so the other mathematicians can agree or challenge, and explain why they do. Is their reasoning the same as the creator’s?
- If they are struggling unproductively, ask “In part b) of the problem ‘Pac(k)-man: Does it fit’, what stopped you from fitting 9 kg, 7 kg, 5 kg, 4 kg, 2 kg and 2 kg into three sacks?” Listen for “It was because the 9 kg had to go on its own and once I’d placed the 7 kg and 2 kg together, the 5 kg could only go with *either* the 4 kg or the 2 kg and so there was one parcel that couldn’t go in a sack.”





Thank you. We hope your mathematicians enjoyed this session.

Please fill out the Sample Session survey by clicking this link [here](#).

If you find anything wrong with these slides, please contact corrections@axiommaths.com