



# Sample Session B

Square squash



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# How can I use the sample sessions?

- Our sample sessions are designed as a taster for Axiom Maths circles. Use them with mathematicians that are interested in participating but seeking more information about the programme before committing.
  - The content and structure is representative of Axiom Maths sessions.
  - The speech bubbles (hidden in presentation view) are included to point out common features of Axiom Maths sessions.
- Sample sessions are designed to be useable with larger groups or whole classes at the beginning of Year 7.
- Training materials that accompany the sample sessions can be found [here](#).

# The ABC of mentoring



## Advance plan

- Tackle the problems yourself and experience the productive struggle
- Consider what the mathematicians will do, say and ask
- Read through the mentor notes and watch the accompanying videos

What are the misconceptions?

What's the same,  
what's different?

What do you  
notice?

## Bring out the maths from them

- Ask questions to extend their mathematical thinking
- Introduce representations to expose the mathematical structure
- ...but give them space to be adventurous. Pause, wait and watch what they do. Get everyone used to an unawkward silence.



## Catalyse conversations

- Sometimes individually, sometimes collectively
- Model and ask for precise mathematical vocabulary
- Help them to help each other
- No child left behind on the mountainside

Are you sure?  
Convince me.

Agree or  
challenge?

What is the correct  
mathematical term for that?



# Modes of engagement

In your '**advance planning**', decide how your circle will engage with the problems so that you '**bring out the maths from them**' and '**catalyse conversations**'. You should consider many things, including the set up of your circle (e.g. online/in person). However, Axiom Maths recommends varying the modes of engagement so that the mathematicians learn to write, think and speak mathematics, both independently and collaboratively.

| Mode                 | Description                                                                                                                                                                                                       | Icon                                                                                  |
|----------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------|
| <b>Independent</b>   | The mathematicians grapple with the problems individually, recording their own efforts. The mentor engages with the mathematicians individually. The mathematicians compare methods with each other as they work. |    |
| <b>Collaborative</b> | The mathematicians work together towards a solution, and create a shared record of their efforts.                                                                                                                 |    |
| <b>Discussion</b>    | The mentor catalyses a conversation between the mathematicians about work that they have done, or are about to do.                                                                                                |   |
| <b>Quick fire</b>    | The mathematicians are time limited in preparing a response to a problem, and show their responses to the mentor and circle simultaneously (e.g. on mini-whiteboards/ in an online 'chat blast')                  |  |



# The intention of the session

- The problems in this session aim to bring out the distinction between “I can’t do it” and “No-one can do it, it’s impossible.” Ask questions such as, “How can you **convince me** that this won’t work whatever way you try to fit the squares in?”
- The mathematicians will develop resilience in the face of problems which they do not solve immediately and will find arguments to *show* that some cases are impossible, rather than just fail to make them work.



# Overview of the session

In this **sample** session, we suggest you may finish the first two 'core' problems in one hour. If you have more or less than this time, use the additional problem for 'rapid graspers' and the guidance below to organise and structure your taster session(s) as you see fit.

| Problem              | Guidance                                                             | You may need...                       |
|----------------------|----------------------------------------------------------------------|---------------------------------------|
| Square squash        | Introduces the context with an impossible problem.                   | Worksheets or <a href="#">Polypad</a> |
| Does it fit?         | Justify whether the squares can be arranged in each given rectangle. |                                       |
| <b>Squash it in!</b> | Additional problem for circles or individuals that progress quickly. |                                       |



# What is a maths circle?

A maths circle is a group of pupils who gather with a mentor to be mathematicians together!

In our **maths circle** we

- **grapple** with (but not always solve!) intriguing problems and ideas
- learn to think, speak and work like a **university mathematician**
- sometimes think on **our own**, sometimes think **in a team**
- enjoy getting **stuck** as well as getting **unstuck**
- have **fun** ourselves, and make it fun for everyone else too!



Reinforce that they will be working on maths problems *together*. In maths circles it's not about competing against each other!

# Good mathematicians are...



**Adventurous**



**Articulate**



**Accepting**

In maths circles you'll learn to be **adventurous**, **articulate** and **accepting** mathematicians!

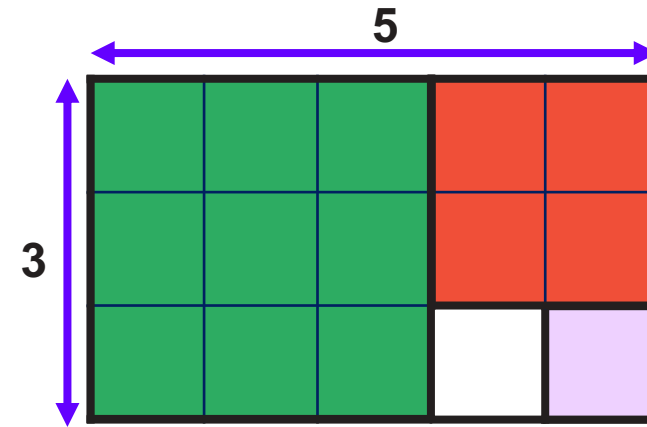
The first three sessions in Block 1 each focus on one of the three **AAA** behaviours of mathematicians.

# Square squash

The diagram shows how the squares with sides 1, 2 and 3 can fit **without overlapping** in a  $3 \times 5$  rectangle.

There is one square left uncovered.

Is there any **smaller rectangle** with whole-number side lengths which will contain **all three** squares?



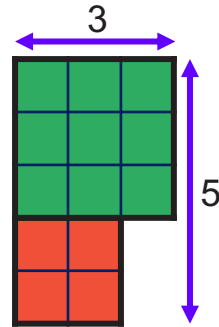
**One square left uncovered**

Axiom Maths problems are designed to provide opportunities for mathematical reasoning in simple contexts which are not heavily dependent on the mathematicians' prior knowledge.

# Square squash

## Look and listen for...

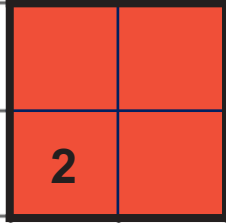
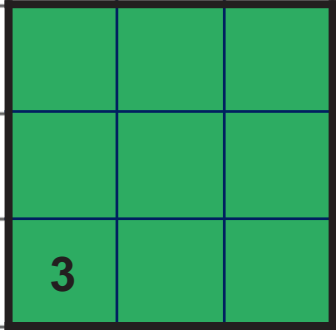
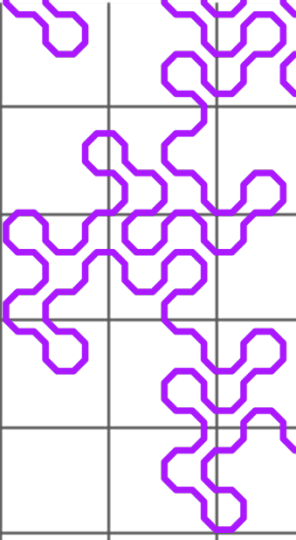
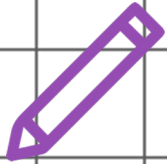
- The mathematicians arranging the squares in various ways, but not successfully reducing the size of rectangle that they fit into.
- “I have to place the  $2 \times 2$  square next to the  $3 \times 3$  square in some way. That means that either the length or width of the rectangle has to be at least 5. And because the  $3 \times 3$  has to fit in the rectangle, the other dimension has to be at least 3, so I can’t get smaller than  $5 \times 3$ .”
- “But wait, the reason is simpler than that! The total area of the squares is  $1^2 + 2^2 + 3^2 = 14$ , so they can’t fit in a rectangle with area less than 14. Only a  $1 \times 14$  rectangle or a  $2 \times 7$  rectangle have area 14, and the  $3 \times 3$  square can’t fit into either of these.”



## Bring out the maths from them

- **This problem is impossible. The purpose is for the mathematicians to *justify why the problem is impossible*.** Don’t rush them to this conclusion. Give them time to be **adventurous** independently first. 
- Whether working online or in person, give the mathematicians squares that they can move around, rather than drawing them each time.
- To bring out at least one of the approaches, ask “What could a rectangle with a smaller area look like?” Listen for
  - “It would have to have width less than 3 or length less than 5.”
  - or “It could have area 14 or 13 or... .”
- “Have you been able to use a smaller rectangle? Do you think it’s possible?  **Convince me!**” You can use the grid on the next slide to support these group discussions.
- **If the detailed ‘area reasoning’ justification does not arise here, wait until a) of the next problem before bringing it out from them.**

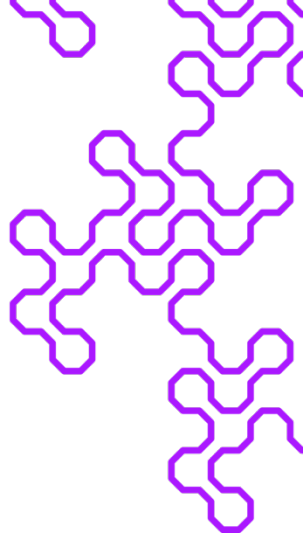
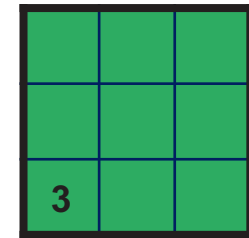
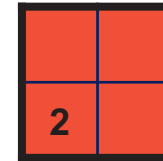
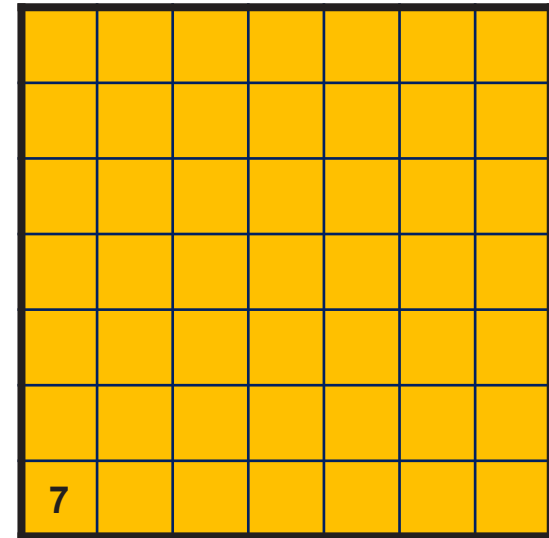
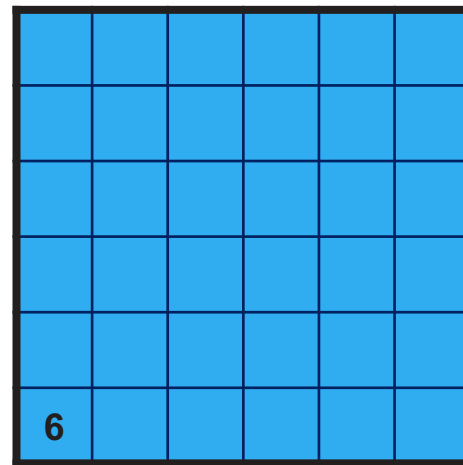
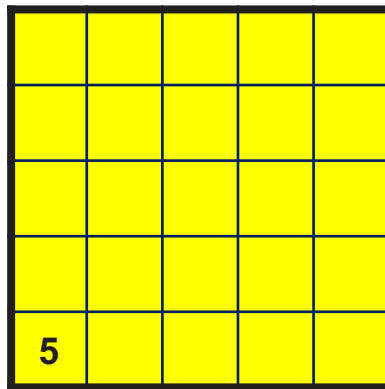
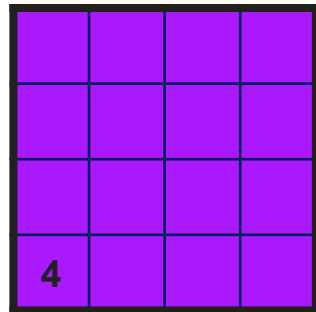




Ask one (or more) of your mathematicians to explain why they think this problem is impossible. Encourage them to visually illustrate their explanations on this slide to support their reasoning.

# Does it fit?

- a) Explain why it's **impossible** to fit squares with sides 1, 2, 3, 4, 5, 6 and 7 in an  $8 \times 17$  rectangle.



# Does it fit?

## Look and listen for...

- The mathematicians re-arranging their squares and trying to fit them into the  $8 \times 17$  rectangle.
- “I worked out the sum of the areas of the squares first.  
 $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 = 140$ .  
But the area of the  $8 \times 17$  rectangle is only  $8 \times 17 = 136$ , so the squares can't possibly fit. There's not enough room!”

## Bring out the maths from them

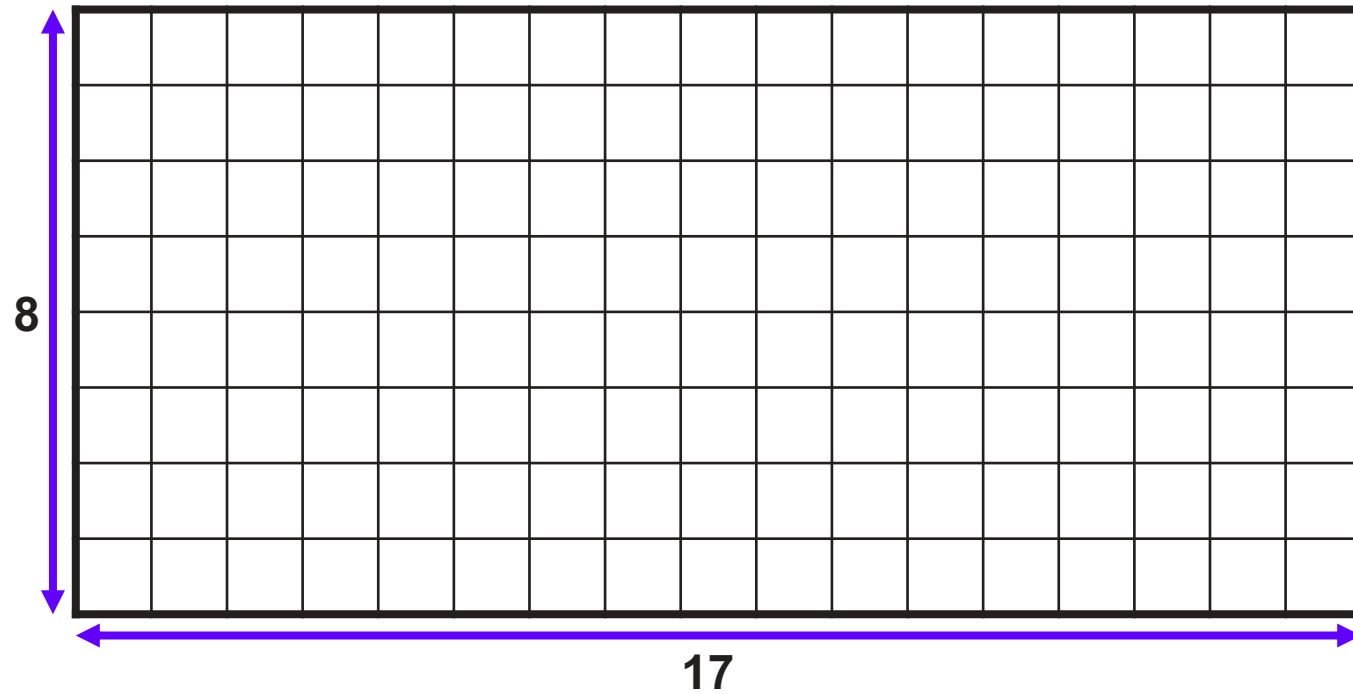
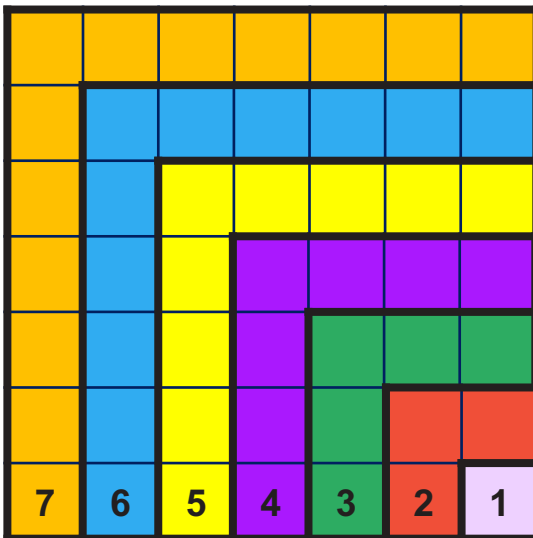
- If the mathematicians say “It's impossible, I've tried and I can't fit the squares in”, ask “How can you be *sure* no one else will be able to fit them in?”
- Encourage them to work systematically. Ask “Which square are you trying to fit in first? Where are you choosing to put it?” Use the next slide to support these  discussions.
- “Did anyone justify why the squares will not fit into the  $8 \times 17$  rectangle differently?”
- **If it doesn't come from them, bring out the ‘area reasoning’ justification here: if the area of the rectangle is less than the sum of the areas of the squares then it's not possible for the squares to fit.** Ask “Can you tell me something about the areas of the squares that is relevant here?”





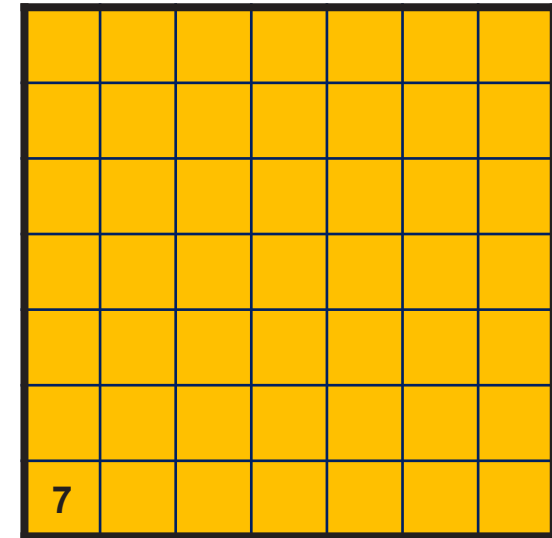
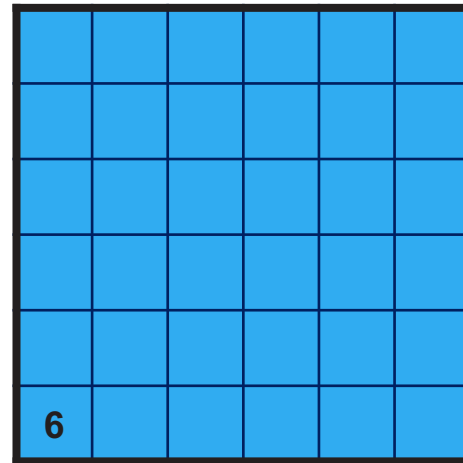
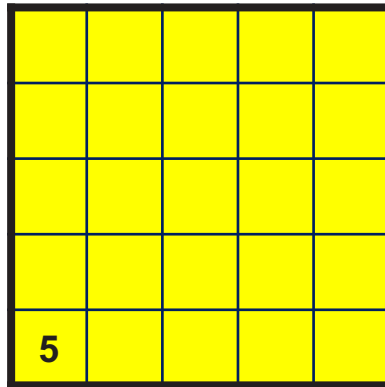
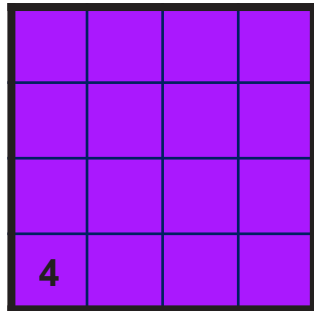
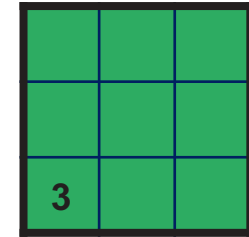
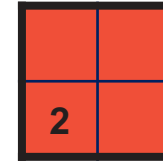
# Does it fit?

- a) Explain why it's **impossible** to fit squares with sides 1, 2, 3, 4, 5, 6 and 7 in an  $8 \times 17$  rectangle.



# Does it fit?

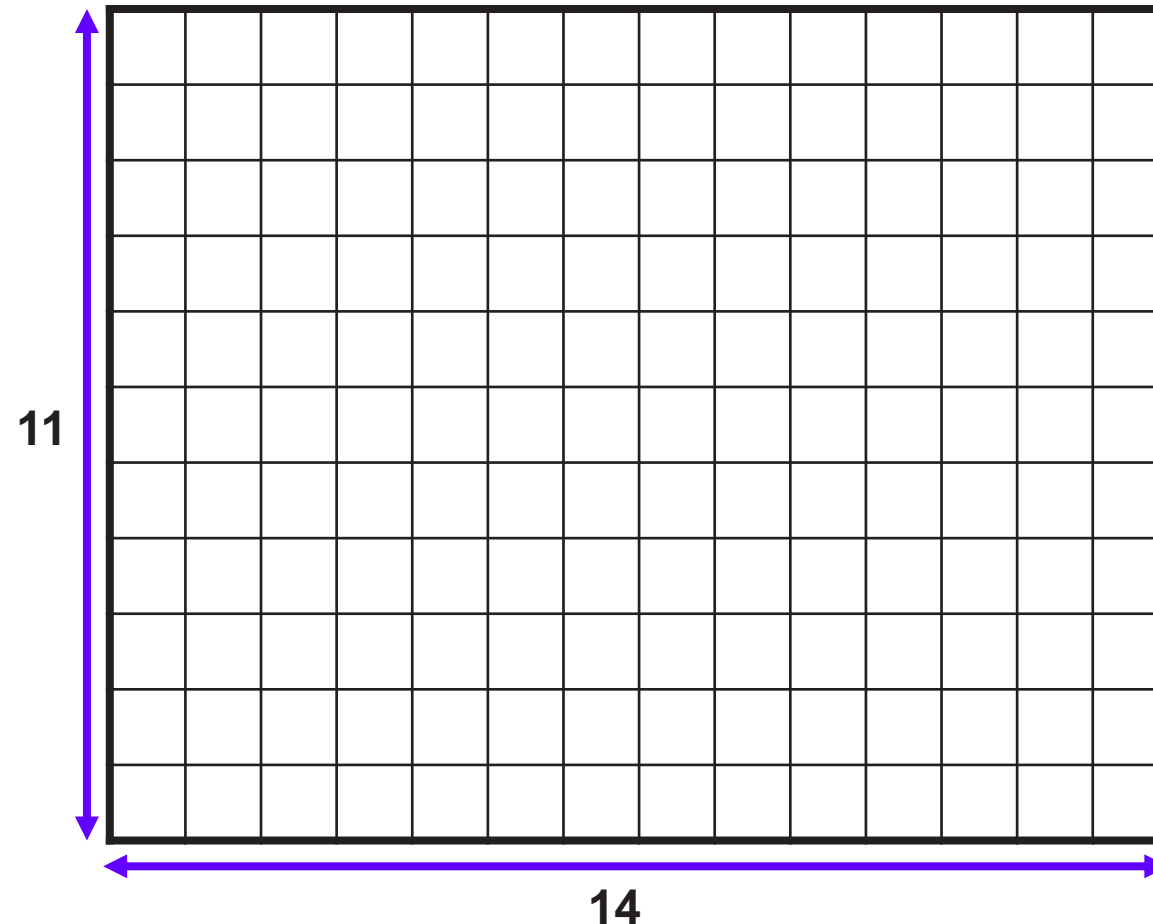
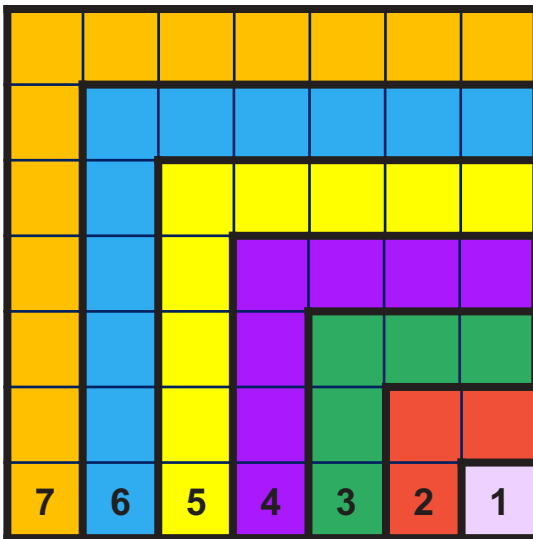
- a) Explain why it's **impossible** to fit squares with sides 1, 2, 3, 4, 5, 6 and 7 in an  $8 \times 17$  rectangle.
- b) Show that squares with sides 1, 2, 3, 4, 5, 6 and 7
- **will fit** in an  $11 \times 14$  rectangle
  - but **will not** fit in a  $10 \times 17$  rectangle.





# Does it fit?

- b) Show that squares with sides 1, 2, 3, 4, 5, 6 and 7 **will fit** in a  $11 \times 14$  rectangle but **will not fit** in a  $10 \times 17$  rectangle.



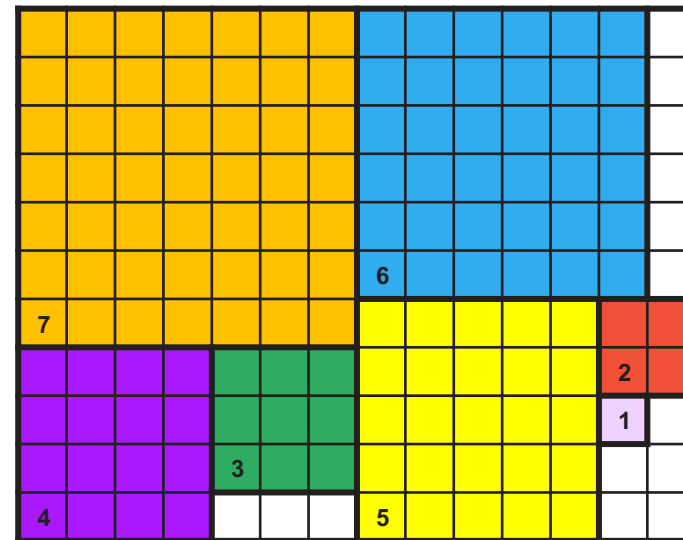
# Does it fit?

## Look and listen for...

- The mathematicians justifying that they *will fit* into the  $11 \times 14$  rectangle by:
  - Finding at least one example where the squares fit. Listen for “I started with the largest squares, as they are the hardest to fit in. Then I fitted the smaller squares around them. I had lots of choices.” Praise the mathematicians for taking a systematic approach. One example is shown here (see right).
- Or by using ‘area reasoning’.
  - “The sum of the areas of the squares is 140, so I think they will fit into an  $11 \times 14$  rectangle because the area of the rectangle is 154.”
  - Now ask if they are *certain* that the squares will fit in and listen for “I don’t know for sure that they will be able to. I’ll try it out and see... .”
  - And then “... ah yes! It works.”

## Bring out the maths from them

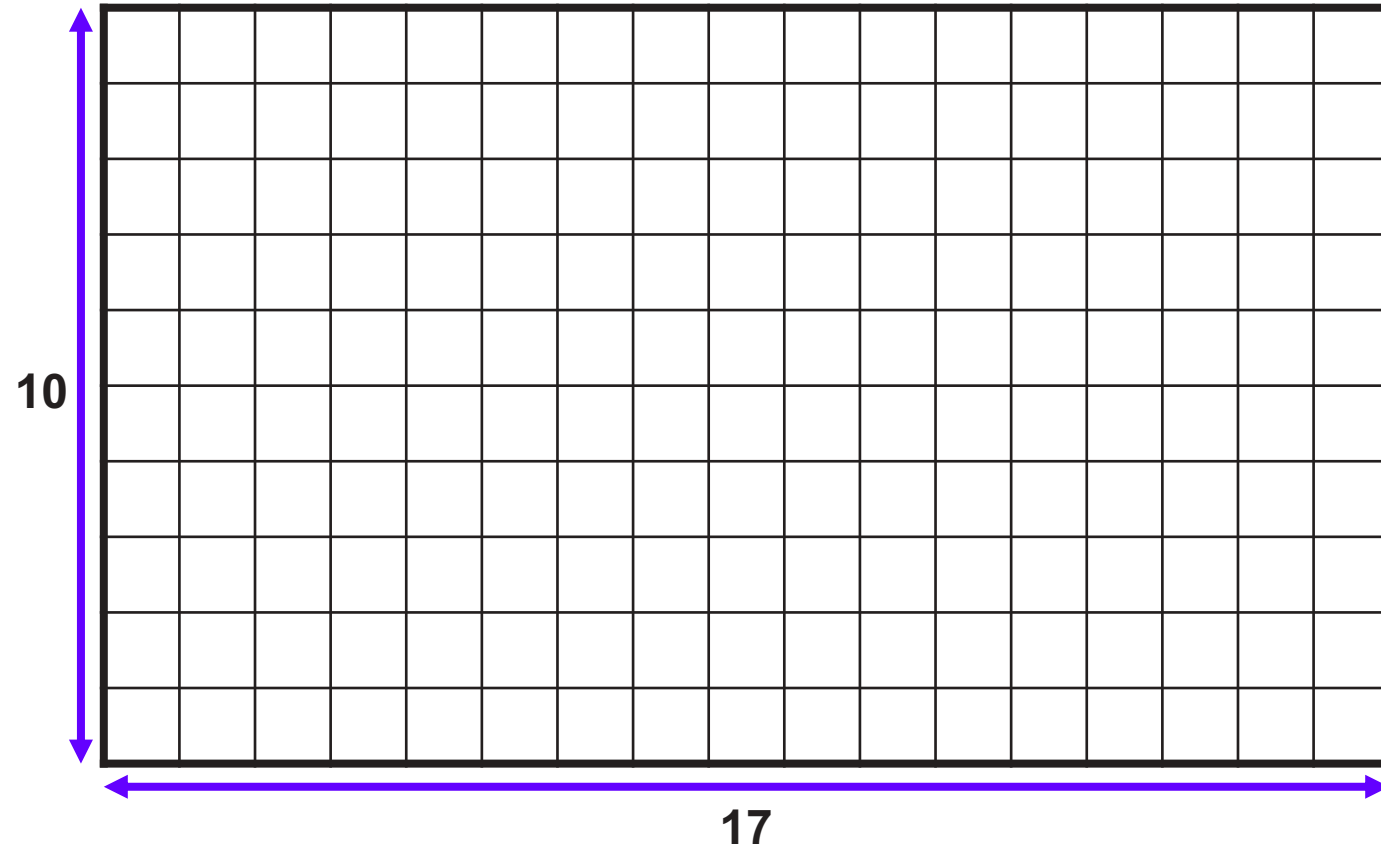
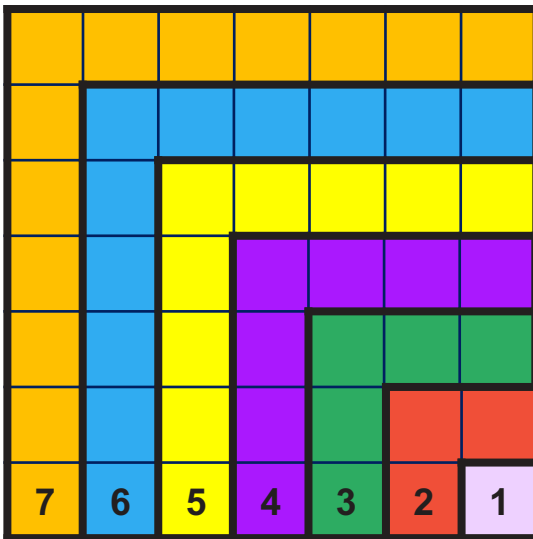
- Bring out that it is not correct to claim that “The area of the rectangle is greater than the sum of the areas of the squares so the squares *must* fit.”. They need to find an example. **Catalyse conversations** by asking “Is ‘area reasoning’ enough to show that the squares *must* fit?” 





# Does it fit?

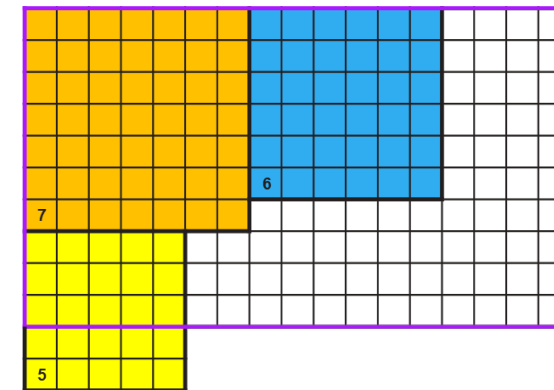
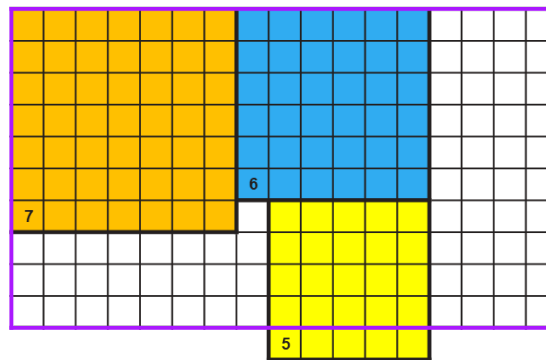
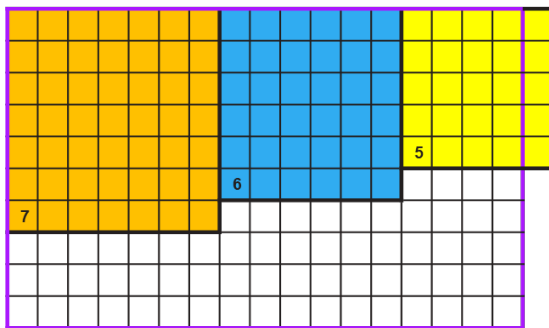
- b) Show that squares with sides 1, 2, 3, 4, 5, 6 and 7 **will fit** in a  $11 \times 14$  rectangle but **will not fit** in a  $10 \times 17$  rectangle.



# Does it fit?

## Look and listen for...

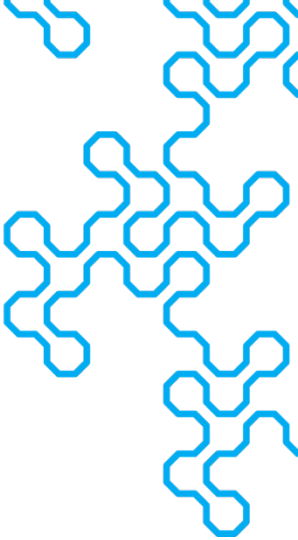
- The mathematicians justifying that they *will not* fit into the  $10 \times 17$  rectangle by:
  - Trying to arrange the squares of length 7 and 6 in a way that ‘leaves room’ for the square of length 5 and then showing that there is no way to make this work.
  - Or by using ‘area reasoning’. Listen for “The sum of the areas of the squares is smaller than the area of the rectangle, so it *might be* possible to fit the squares in... I don’t know for sure though. I’ll need to try it out and see...”
  - ...and then “Uh-oh. This time it’s not possible! There’s no way that I can fit the squares of lengths 7, 6 and 5 into the rectangle because  $7 + 6 + 5$  is more than 17, and  $7 + 6$  and  $6 + 5$  and  $7 + 5$  are all more than 10.”



# Does it fit?

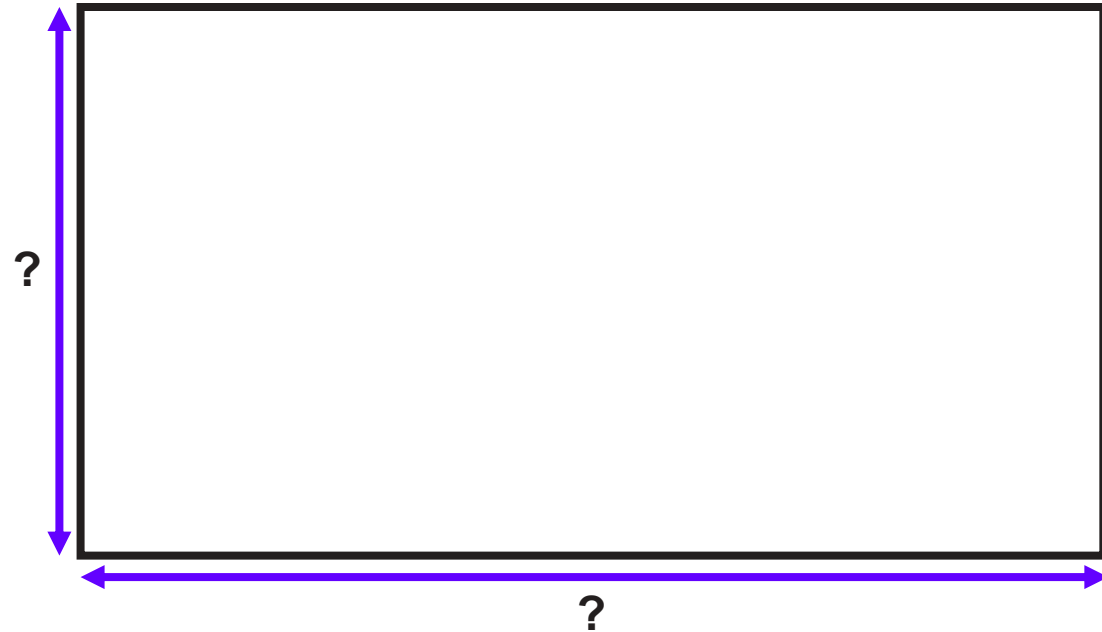
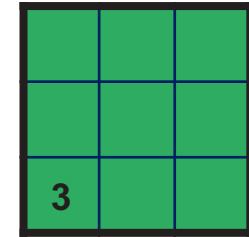
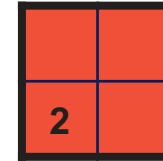
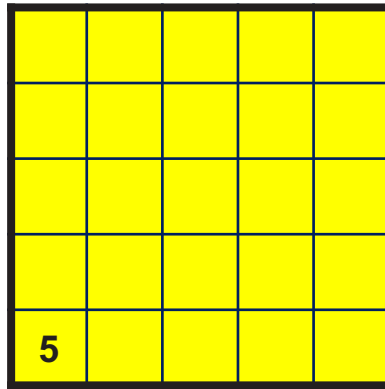
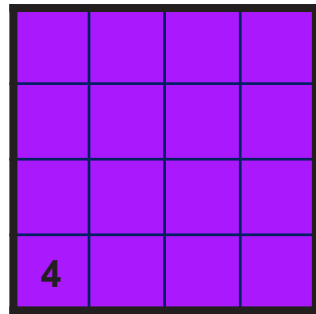
## Bring out the maths from them

- “Can you tell me something about the values of the lengths of the squares that is stopping you from fitting them all in?”
- **Ask the mathematicians to reflect on what they have learned and to articulate what the ‘area reasoning’ tells them.** Ask “If the area of a rectangle is greater than the sums of the areas of the squares, what does this tell you? What doesn’t it tell you?” Listen for:
  - “It tells you that it *might be* possible for the squares to fit because the area of the rectangle cannot be less than the sum of the areas of the squares.”
  - “But it doesn’t tell you that it *must* be possible. You need to find an example to be certain.”



# Does it fit?

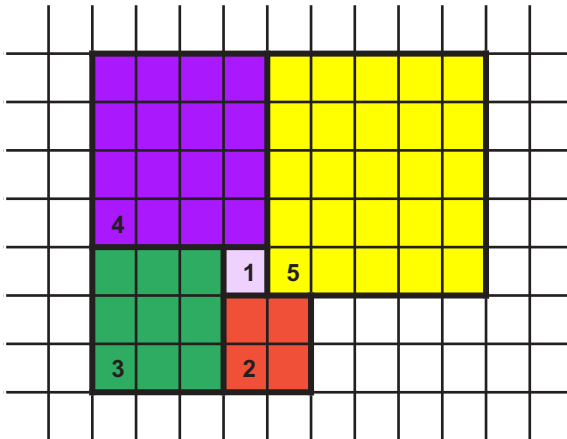
- c) Find the **smallest** rectangle with whole-number side lengths in which squares with sides 1, 2, 3, 4 and 5 **will fit** without overlapping.



# Does it fit?

## Look and listen for...

- The mathematicians ‘building’ a big-enough rectangle by arranging the squares and drawing a rectangle which contains the squares:
  - “I placed the square with length 5 next to the square with length 4. Then I had a few choices of where to put the square with length 3.”
  - “Look, I can fit them into a  $7 \times 9$  rectangle that has area 63.”



## Bring out the maths from them

- Give the mathematicians time to be **adventurous** and to work in  **small groups to come up with their own solutions before prompting them.**
- Promote collaboration by bringing them back together to discuss their solutions. If they have been successful with this ‘building’ method, ask “Are you *sure* that you can’t fit the squares into a smaller rectangle? **Convince me.**” 
- They can rule out smaller possibilities by drawing rectangles with smaller areas and justifying why they can’t contain the squares. Bring this out using the prompts on the next slide.



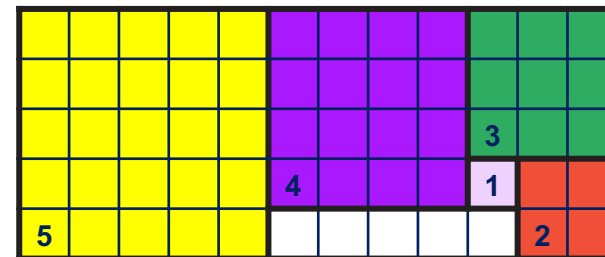
# Does it fit?

## Look and listen for...

- The mathematicians working systematically starting from the smallest possible area.
  - “The sum of the areas of the squares is 55 so the rectangle must have area at least 55. I can’t use a  $5 \times 11$  rectangle because  $5 + 4 + 3$  is more than 11.”
  - “I’ll try a rectangle of area...
    - ...56. This can be  $7 \times 8$  or  $14 \times 4$  or  $28 \times 2$  or  $56 \times 1$ . Only  $7 \times 8$  will contain the square of length 5 but I can’t also fit in the square of length 4.”
    - ...57. This will be far too ‘narrow’. Its only factor pairs are  $1 \times 57$  and  $3 \times 19$ . I can’t fit anything bigger than the square of length 3 into these.”
    - ...58.  $1 \times 58$  or  $2 \times 29$  work but both are too ‘narrow’. 58 doesn’t have a factor pair in which both factors are 5 or greater.”
    - ...59. This is worse because 59 is a prime number! The only possibility is  $1 \times 59$ , so I can only fit in the square of length 1.”
    - ...60. Hmm, I can’t make  $6 \times 10$  work. I’ll try  $5 \times 12$  ... yes, this works!” (see right)

## Bring out the maths from them

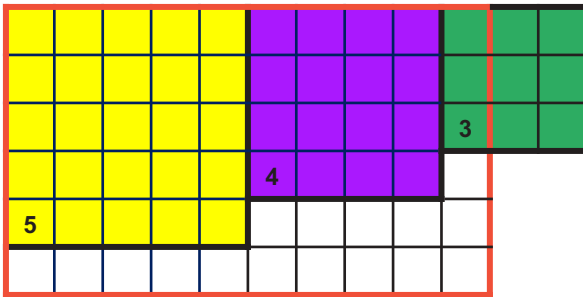
- **The purpose is for them to apply what they learned in a) and b) to solve this problem systematically starting from the smallest possible area.** If necessary, ask:
  - “What is the smallest area the rectangle can be for the squares to fit? What can you tell me about the dimensions of the rectangle?”
  - “Why can’t you have a  $[P \times Q]$  rectangle?”
  - When they get to 59, ask “How can you tell without trying that the squares won’t fit into a rectangle with an area of 59?” Give some thinking time. If necessary, ask “What can you tell me about the factors of 59?”



# Does it fit?

## Look and listen for...

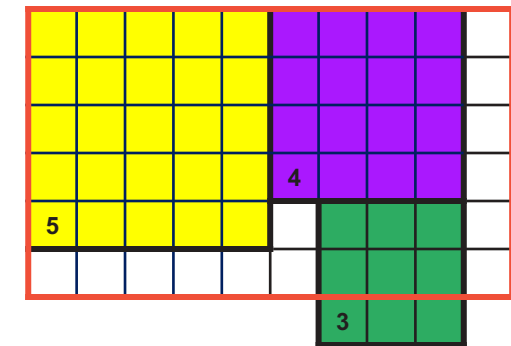
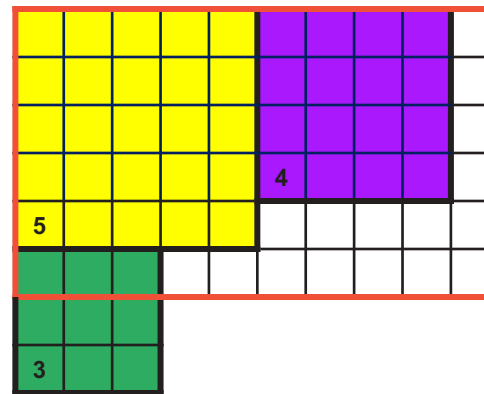
- After they've solved the problem, "I now know that  $5 \times 12$  works, but I wonder why  $6 \times 10$  didn't work. I'll investigate... Wherever I put the squares with lengths 4 and 5 in the rectangle, there's no space for the square with length 3. The problem is that  $5 + 4 + 3$  is more than 10 (see below) and also both  $5 + 3$  and  $4 + 3$  are more than 6." (see right).



- "So even though the smallest rectangle where the squares fit has area 60, it doesn't mean that the squares will fit into *any* rectangle with area 60. The  $5 \times 12$  is important to specify."

## Bring out the maths from them

- Ask for an **articulate** explanation (drawn or verbal) why the squares won't fit into a  $6 \times 10$  rectangle, which *a*lso has area 60.
- You will likely need to bring out this curiosity, and reiterate to them that '**the answer is only the beginning!**'



# Today, you have you been...



## Adventurous

After failing to fit the squares in the rectangle a few times, I tried a different strategy and used 'area reasoning' instead.



## Articulate

I explained to my partner that 'area reasoning' only tells you that it *might* be possible for the squares to fit.



## Accepting

I asked my partner what strategy they used when they fitted the squares with sides 1, 2, 3, 4 and 5 into a smaller rectangle than me.

At the end of each Axiom Maths session (as well as throughout), the mathematicians reflect on how they have shown **AAA** behaviour in this session!

# Today, you have you been...

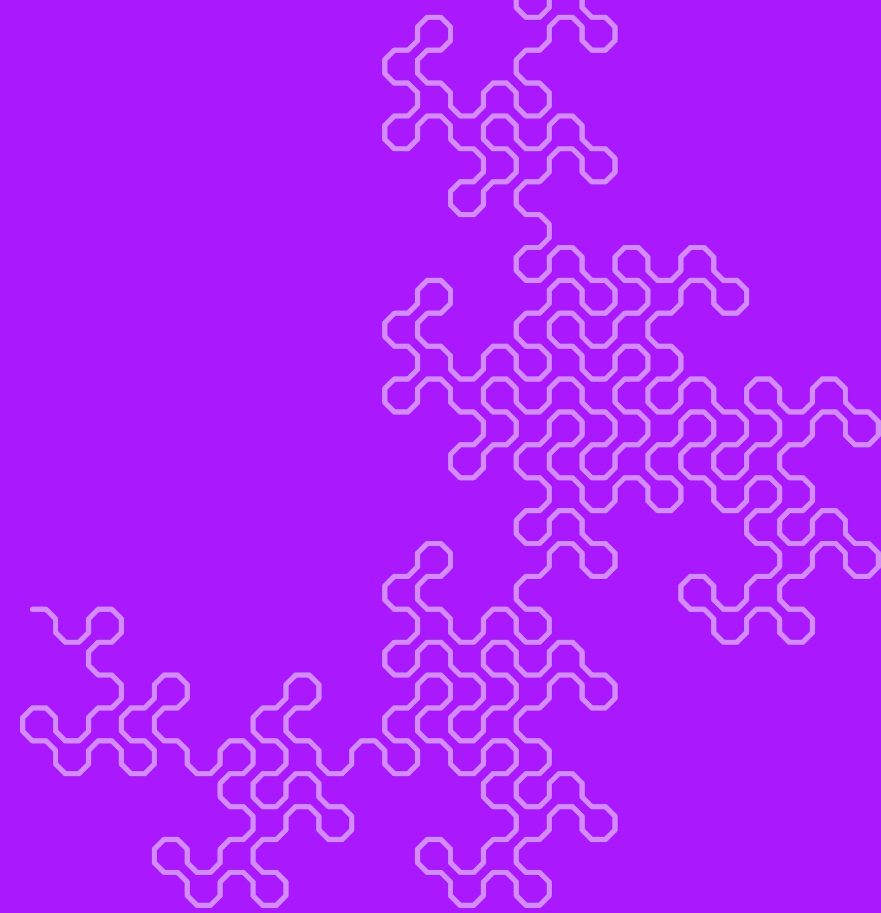
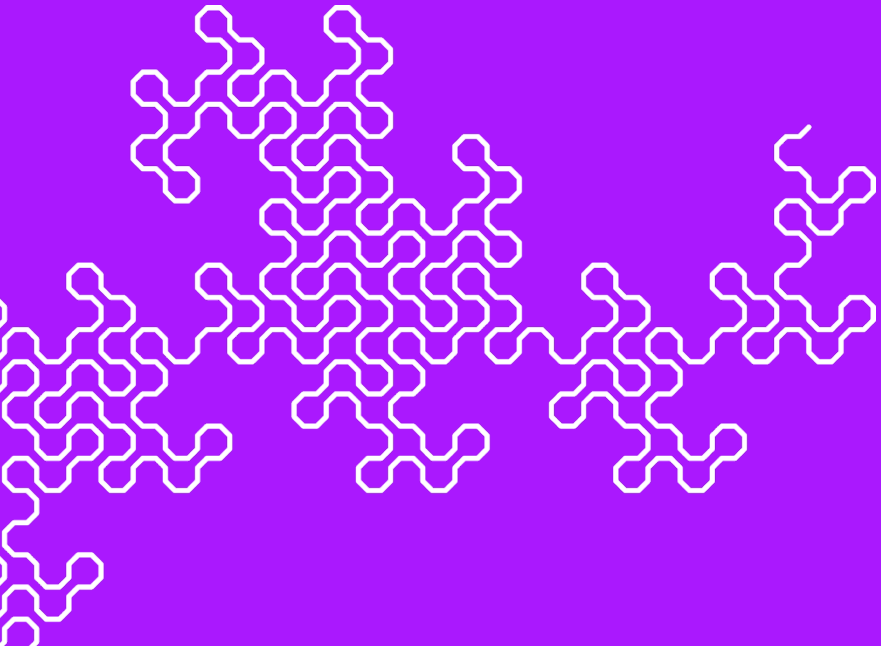
## Bring out the maths from them

- **Allow 2 minutes at the end of the session to tell the mathematicians how they have demonstrated the three AAA behaviours of mathematicians today.**
- Ask them to reflect on what they enjoyed about this session, and how it might have been different from a normal maths lesson.
- “Is there somebody who you think has been especially **adventurous/articulate/accepting** today? What did they do?”

The first three sessions in Block 1 each focus on one of the three **AAA** behaviours of mathematicians.

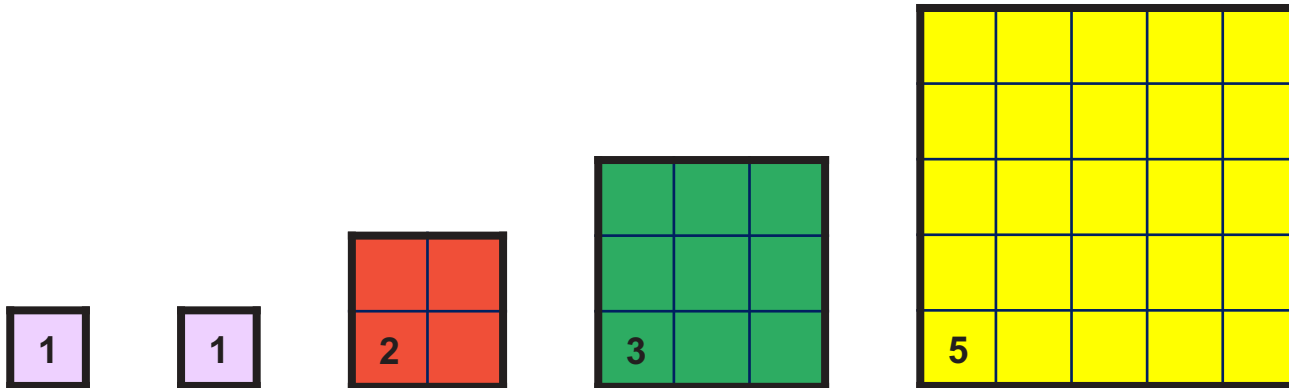


# Problems for rapid graspers



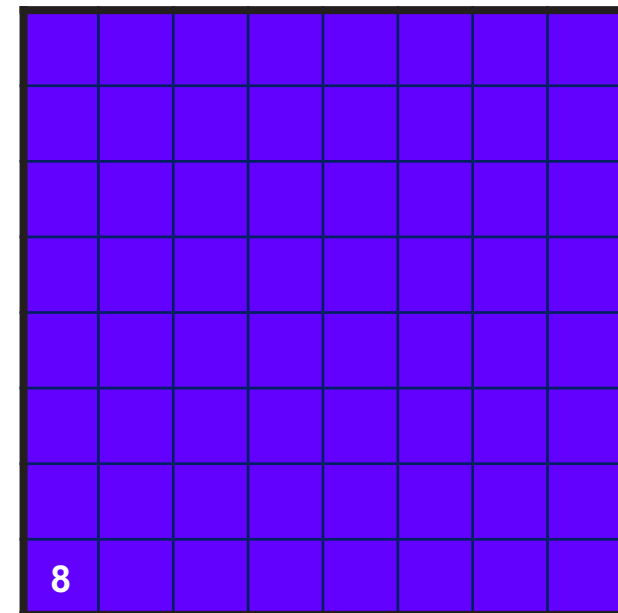
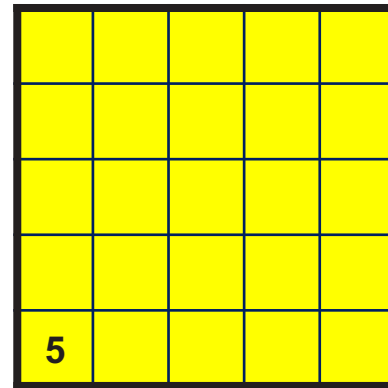
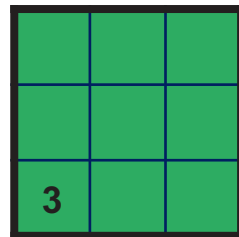
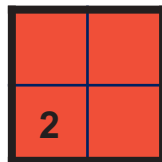
# Squash it in!

- a) Can squares with sides 1, 1, 2, 3, and 5 fit **exactly** into a rectangle, with no squares left uncovered?



# Squash it in!

- a) Can squares with sides 1, 1, 2, 3, and 5 fit **exactly** into a rectangle, with no squares left uncovered?
- b) Can squares with sides 1, 1, 2, 3, 5 and 8 fit **exactly** into a rectangle, with no squares left uncovered?

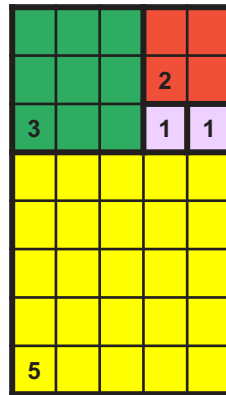


# Squash it in!

## Look and listen for...

For a)

- “I’ll work out the sum of the areas of the squares first.  $1^2 + 1^2 + 2^2 + 3^2 + 5^2 = 40$ . The factor pairs of 40 are  $1 \times 40$ ,  $2 \times 20$ ,  $4 \times 10$  (which are too small) and  $5 \times 8$ .”
- Or the mathematicians ‘building’ a big-enough rectangle by arranging the squares and drawing a rectangle which contains the squares.
- “I can fit them into a  $5 \times 8$  rectangle that has area 40.”

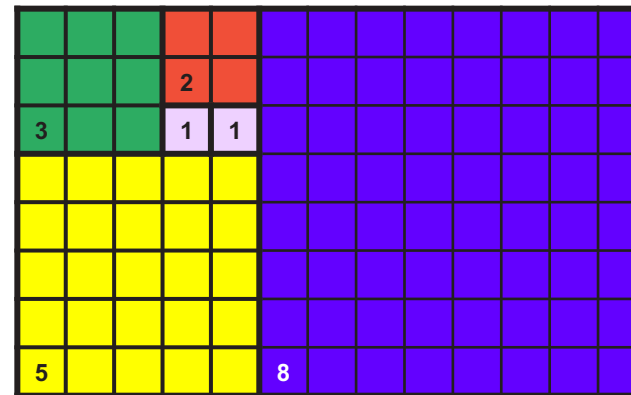


For b)

- “I can add the square with length 8 onto the side of my rectangle from part a) to form an  $8 \times 13$  rectangle that has area 104.

## Bring out the maths from them

- “Are your solutions unique?” Listen for “Yes, because 40 and 104 don’t have any other pairs of factors which would make rectangles that could fit the largest square inside.”
- “What size squares could you add next?” Listen for
  - “A square with length 13 which would make a  $13 \times 21$  rectangle.”
  - “Then I could add square with length 21... wait! I recognise this sequence of numbers!”



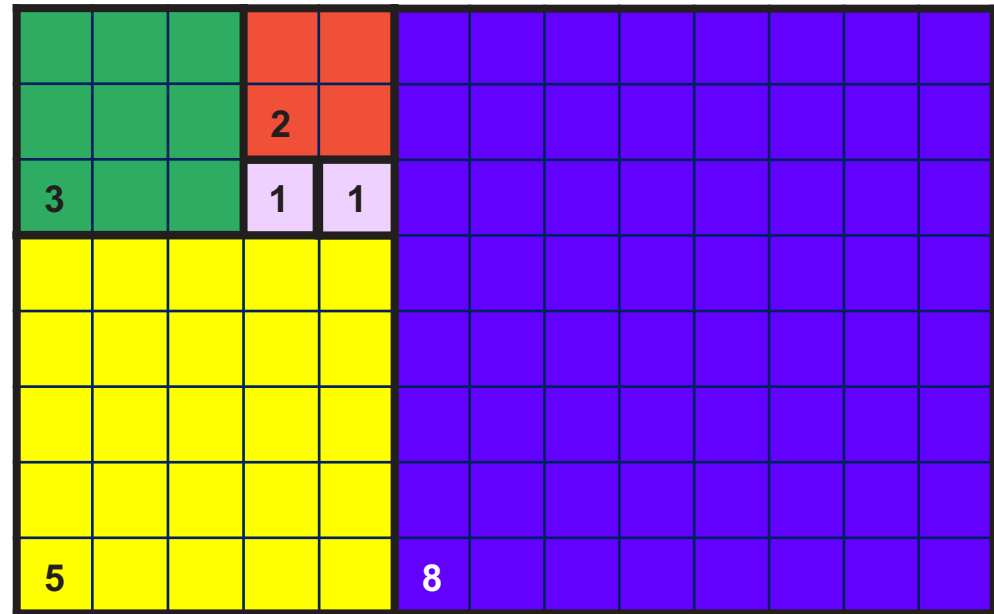
# Squash it in!

The **Fibonacci numbers** are the numbers in this list.

1, 1, 2, 3, 5, 8, ...

Each number in the list is the **sum of the two previous numbers** in the list.

- c) Will squares with side lengths that are Fibonacci numbers **always** fit **exactly** into a rectangle, with no squares left uncovered?



# Squash it in!

## Look and listen for...

- “Each rectangle’s width and length are two consecutive Fibonacci numbers. For example,  $5 \times 8$  or  $8 \times 13$ .”
- “That means that the sum of the width and length is the next Fibonacci number after that.”
- “Then I add a square on the long side of the rectangle.”
- “Now, the new width is the old length, which is a Fibonacci number.”
- “Also, the new length is the sum of the old width and length. And this equals the next Fibonacci number!”
- “So squares with side lengths that are Fibonacci numbers will always fit exactly into a rectangle, with no squares left uncovered. I could keep adding squares forever!”

## Bring out the maths from them

- The aim is for the mathematicians to move from “it works for these cases” to “it will always work, and here’s why.”
- If the mathematicians are struggling unproductively, ask
  - “What do you notice about the dimensions of the rectangles?”
  - “When you add the next square, how does the rectangle change?”
- **“What about if the smallest square has side length 2? Do the squares fit exactly into a rectangle then?”** Listen for “No, it only works if I start at the beginning of the sequence, (1, 1, 2, ...) and use every Fibonacci number in order without skipping any.”





**Thank you. We hope your mathematicians enjoyed this session.**

**Please fill out the Sample Session survey by clicking this link [here](#).**

**If you find anything wrong with these slides, please contact [corrections@axiommaths.com](mailto:corrections@axiommaths.com)**